Planets and exoplanets

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Summary

This workshop provides a series of activities to compare the many observed properties (such as size, distances, orbital speeds and escape velocities) of the planets in our Solar System. Each section provides context to various planetary data tables by providing demonstrations or calculations to contrast the properties of the planets, giving the students a concrete sense for what the data mean. As a final activity, some properties of extrasolar planetary systems are explored and compared to the Solar system. At present, several methods are used to find exoplanets, more or less indirectly. It has been possible to detect almost 100 multiple planetary systems. A famous example is shown in figure 1.

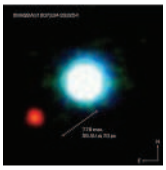


Fig. 1: The first planet directly observed 2M1207b. It has a mass 3.3 times the mass of Jupiter and orbits at 41 AU from the brown dwarf. In 2006, a disk of dust was found around the parent star, providing evidence that planet formation may proceed in a way similar to that observed around more massive solar-type stars. (Photo: ESO).

Goals

• Understand what the numerical values in the Solar Sytem summary data table mean.
• Deduce the orbital radii and orbital periods of the Galilean satellites of Jupiter using a set of photographic observations.
• Calculate Jupiter's mass using Kepler's third law.
• Understand the main characteristics of extrasolar planetary systems by comparing their properties to the orbital system of Jupiter and its Galilean satellites.

Solar System and date tables

By creating scale models of the solar system, the students will compare the different planetary parameters. To perform these activities, we will use the data in Table 1.

In all cases, the main goal of the model is to make the data understandable. Millions of kilometers are not distances that are easily grasped. However, if translated to scaled distances and sizes, the students usually find them easier to comprehend.

Planets	Diameter (km)	Distance to Sun (km)
Sun	1,392,000	
Mercury	4,878	$57.9\ 10^6$
Venus	12,180	$108.3\ 10^6$
Earth	12,756	$149.7\ 10^6$
Marte	6,760	$228.1\ 10^6$
Jupiter	142,800	$778.7\ 10^6$
Saturn	120,000	$1,430.1\ 10^6$
Uranus	50,000	$2,876.5\ 10^6$
Neptune	49,000	$4,506.6\ 10^6$

Table 1: Data of the Solar System bodies.

Model of the Solar System

Models of diameters

Using a large piece (or multiple pieces if necessary) of yellow paper cut a circle representing the Sun. The Sun is scaled to be 139 cm in diameter such that 1 cm is 10 000 km. Cut the different planets out of plain cardboard or construction paper and draw their morphological characteristics. By placing the planets near the solar disk, students can grasp the different planetary scales.

With a scale of 1 cm per 10 000 km, use the following planetary diameters:

Sun 139 cm, Mercury 0.5 cm, Venus 1.2 cm, Earth 1.3 cm, Mars 0.7 cm, Jupiter 14.3 cm, Saturn 12.0 cm, Uranus 5.0 cm and Neptune 4.9 cm.

Suggestion: It is also possible to complete the previous model by painting the planets on a shirt, keeping the scale of the planets but only painting a fraction of the Sun.

Model of distances

By comparing the distances between the planets and the Sun we can produce another model that is easy to set up in any school hallway. First, simply cut strips of cardboard 10cm wide, linking them up to obtain a long strip of several meters (figure 3). Then, place the

Fig. 2a and 2b: xamples of shirts providing Solar and planetary diameter scale comparisons.

cutouts of the planets on it at their correct distances. Remind the students that the distance between the planets are not to scale with diameters. At the suggested scale, the planets would be one thousand times smaller as here we are using 1 cm per 10 000 000 km, while in the first activity above we used 1 cm per 10 000 km. If using a scale of 1cm per 10 million km the scaled distances are: Mercury 6 cm, Venus 11 cm, the Earth 15 cm, Mars 23 cm, Jupiter 78 cm, Saturn 143 cm, Uranus 288 cm and Neptune 450 cm.

Fig. 3: Model of distances.

Suggestion: A fun variation of this model is to use a toilet paper roll each sheet for scale. For example, you can take as scale a portion of paper for every 20 million km.

Model of diameters and distances

The next challenge is to combine the two above activities and make a model representing the bodies to scale, as well as the corresponding distances between them. It is not actually that easy to define a scale that allows us to represent the planets with objects that are not too small and still have distances that are not overly large, in which case the sizes and distances are not easily assimilated, and the model is not very useful for students. As a suggestion, it may be a good idea to use the schoolyard to make the model and use balls for the planets as balls of varying diameters are available as appropriate.

As an example, we provide a possible solution. At one end of the schoolyard we put a basketball about 25 cm in diameter that represents the Sun. Mercury will be the head of a needle (1 mm in diameter) located 10 m from the Sun. The head of a slightly larger needle (2 mm in diameter) will represent Venus at 19 m from the Sun, while Earth will be the head of another needle similar to the previous one (2 mm) at 27 m from the Sun. Mars is a slightly smaller needle head (1mm), located 41 m from the Sun. Usually, the schoolyard ends here, if not sooner. We will have to put the following planets in other places outside the schoolyard, but at landmarks near the school, so that the students are familiar with the distances. A ping-pong ball (2.5 cm diameter) corresponds to Jupiter at 140 m from the Sun. Another ping-pong ball (2 cm in diameter) will be Saturn at 250 m from the Sun, a glass marble (1 cm in diameter) will represent Uranus at 500 m from the Sun, and a final marble (1 cm), located at 800 m, will represent Neptune.

It should be emphasized that this planetary system does not fit into any school. However, if we had reduced the distances, the planets would be smaller than the head of a needle and would be almost impossible to visualize. As a final task, you can calculate what scale has been used to develop this model.

Fig. 4: The Sun and the planets of the model of diameters and distances.

Model on a city map

The idea is simple - using a map of the city to locate the positions of the different planets, assuming the Sun is located at the entrance to the school. As an example, we present the map of Barcelona with different objects (specifically fruits and vegetables) that would be located on the different streets, so you can better imagine their size. As an exercise, we suggest that you do the same activity with your own city.

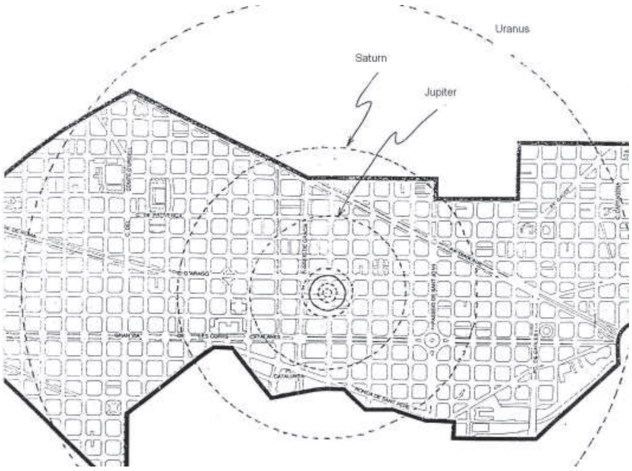


Fig. 5: Map of the "Ensanche de Barcelona" with some planets.

Using the map shown here, Mercury would be a grain of caviar, Venus and the Earth a couple of peas, Mars a peppercorn, Jupiter an orange, Saturn a tangerine and Uranus and Neptune a pair of walnuts. For the Sun, since there is no vegetable large enough, students should imagine a sphere roughly the size of a dishwasher. The instructor can do the same activity using their own city.


Fig. 6a and 6b: Snapshots of the city of Metz.

In the city of Metz (France) there is a solar system spread out on its streets and squares, with corresponding planets accompanied by information panels for those walking by.

Models of light distances

In astronomy it is common to use the light year as a unit of measurement, which can often be confused as a measurement of time. This concept can be illustrated using a model of the Solar System. Since the speed of light is c = 300,000 km/s., the distance that corresponds to 1 second is 300,000 km. For example, to travel from the Moon to the Earth, which are separated by a distance of 384,000 km, it takes light 384,000/300,000 = 1.3 seconds.

$$\frac{384{,}000}{300{,}000} = 1.3 \text{ seconds}$$

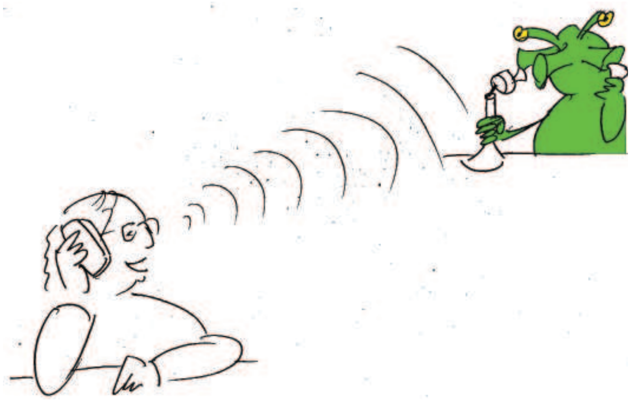


Fig. 7: Another example.

Using these units, we will instruct the students to calculate the time required for sunlight to reach each of the planets of the Solar System. (For the instructor, here are the times required: the time it takes sunlight to reach Mercury is 3.3 minutes, to Venus it takes 6.0 minutes, to Earth 8.3 minutes, to Mars 12.7 minutes, to Jupiter 43.2 minutes, to Saturn 1.32 hours, to Uranus 2.66 hours and to Neptune, 4.16 hours.

You may want to ask the students to imagine what a video conference between the Sun and any of the planets would be like.

Model of the apparent size of the solar disk from each planet

From a planet, for example the Earth, the Sun subtends an angle α (figure 8). For very small values of α, we take tan α = α (in radians)

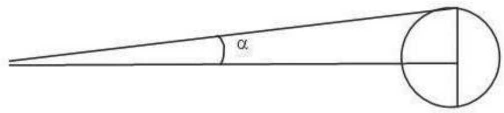


Fig. 8: From the Earth, the Sun subtends an angle α.

Knowing that the solar diameter is $1.4\ 10^6$ km, ie a radius of $0.7\ 10^6$ km, and that the Earth-Sun distance is $150\ 10^6$ km, we deduce:

$$\alpha = \tan \alpha = \frac{0.7 \cdot 10^6}{150 \cdot 10^6} = 0.0045 \text{ radians}$$

And in degrees:

$$\frac{0.0045\ 180}{\pi} = 0.255^o$$

That is, from the Earth, the Sun has a size of 2 x 0.255º=0.51°, i.e., about half a degree. Repeating the same process for each planet, we get the results in the following table 2 and we can represent their relative sizes (figure 9).

Planets	tan α	α (°)	α (°)aprox
Mercury	0.024	1.383	1.4
Venus	0.0129	0.743	0.7
Mars	0.006	0.352	0.4
Jupiter	0.0018	0.1031	0.1
Saturn	0.000979	0.057	0.06
Uranus	0.00048	0.02786	0.03
Neptune	0.0003	0.0178	0.02

Table 2: Results for the different planets.

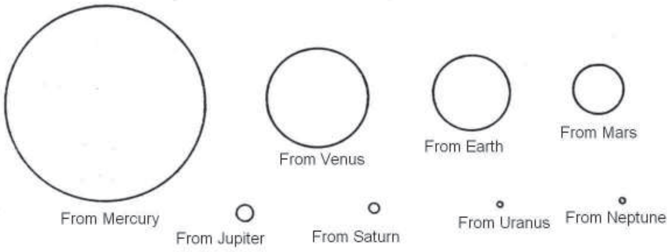


Fig. 9: The Sun seen from each planet: Mercury, Venus, The Earth, Mars, Jupiter, Saturn, Uranus and Neptune.

Model of densities

The objective of this model is to look for samples of materials that are easily manipulated and have a density similar to each of the solar system bodies, in order to be able to "feel it in our hands."

Planets	Density (g/cm^3)
Sun	1.41
Mercury	5.41
Venus	5.25
Earth	5.52
Moon	3.33
Mars	3.9
Jupiter	1.33
Saturn	0.71
Uranus	1.3
Neptune	1.7

Table 3: Densities of the bodies in the Solar System.

From Table 3 of planetary densities, simply compare with the densities of various minerals (in every school there is usually a collection of materials) or with samples of other materials that are easy to find such as glass, ceramics, wood, plastics, etc.. The following Table 4 presents some examples of materials and their densities.

When using materials not included in Table 4, it is very easy to calculate its density. Simply take a portion of this material, weigh it to find its mass, m, and put it in a container of water to measure its volume, V. The

Fig. 10: Model of densities.

Minerals	Density	Other materials	Density
Plaster	2.3	Glycerin	1.3
Orthoclase	2.6	Cork	0.24
Sulfur	1.1-2.2	Aluminium	2.7
Alite	2	Iron	7.86
quartz	2.65	Cement	2.7 - 3.1
Borax	1.7	Glass	2.4 - 2.8
Blende	4	Tin	7.3
Pyrite	5.2	Clay	1.8 - 2.5
Erythro-cytes	5.4	Bakelite	1.25
Calcite	2.7	Oak	0.90
Galena	7.5	Pinewood	0.55

Table 4: Examples of densities of some materials

density d of the material will be,

$$d = \frac{m}{V}$$

Students should notice that Saturn would "float" in water, because its density is less than 1.

Flattening model of planets

To visualize the deformation (flattening) of gas planets due to the centrifugal force generated by their rotation, we will build a simple model.

As we can see in figure 9, with a stick and some cardboard strips, we can build this simple model that reproduces the flattening of Solar System planets due to rotation.

1. Cut some cardboard strips 35 per 1 cm in size.
2. Attach both ends of the strips of cardboard to a 50 cm-long cylindrical stick. Attach the top ends to the stick so that they cannot move, but allow the bottom ends to move freely along the stick.
3. 3. Make the stick turn by placing it between two

hands, then rotating it quickly in one direction and then the other. You will see how the centrifugal force deforms the cardboard bands (figure 11) in the same way it acts on the planets.

Fig. 11: Model to simulate flattening due to rotation.

Model about planetary orbital speeds

It is well known that not all planets orbit the sun with the same speed (table 5).

Planet	Orbital average speed (km/s)	Distance from the Sun (km)
Mercury	47.87	$57.9\ 10^6$
Venus	35.02	$108.3\ 10^6$
Earth	29.50	$149.7\ 10^6$
Mars	24.13	$228.1\ 10^6$
Jupiter	13.07	$778.7\ 10^6$
Saturn	9.67	$1{,}430.1\ 10^6$
Uranus	6.84	$2{,}876.5\ 10^6$
Neptune	5.48	$4{,}506.6\ 10^6$

Table 5: Orbital data of the Solar System bodies.

The fastest is Mercury, the closest, and the slowest is Neptune, the farthest. Romans had already noticed that Mercury was the fastest of all and so it was identified as the messenger of the gods and represented with winged feet. Even if observing with the naked eye, it is possible to tell that Jupiter and Saturn move much more slowly across the zodiacal constellations than do Venus and Mars, for example.

From Kepler's third law $P^2/a^3 = K$, it is deduced that the orbital speed decreases when the distance increases.

To view this relationship, there is also a simple way to experience this relationship. We begin by tying a heavy object, such as a nut, onto a piece of string. Holding the string from the end opposite the heavy object, we spin the object in a circular motion above our heads. We can then see that if we release string as we spin it (making the string longer), the object will lose speed. Conversely, if we take in string (making it shorter), it will gain speed. In fact, this (e.g. Kepler's third law) is a consequence of the conservation of angular momentum.

Fig. 12a, 12b and 12c: Simulating the circular movement of planets.

We can then develop a solar system model with nuts and bits of string proportional in length to the radii of the planetary orbits (assuming, again, that they all travel in circular orbits). However, instead of cutting a separate piece for each planet, cut all pieces to a length of about 20 cm. Then, using the appropriate scaling, measure the correct distance from the heavy object and make a knot at this point. Then, the string can be held at the location of the knot while spinning the heavy object.

To use the model we must hold one of the strings at the location of the knot and turn it over our heads in a plane parallel to the ground with the minimum speed possible speed that will keep it in orbit. We will see that this velocity is greater in cases where the radius is smaller.

Model of surface gravities

The formula for gravitational force,

$$F = G \cdot \frac{M \cdot m}{d^2}$$

allows us to calculate the surface gravity that acts on the surface of any planet. Considering a unit mass (m = 1) on the planet's surface (d = R), we obtain

$$g = \frac{G \cdot M}{R^2}$$

If we then substitute $M = 4/3\ \pi R^3\ \rho$, for the planet mass, we find:

$$g = \frac{4}{3}\pi \cdot G \cdot \rho \cdot R$$

where $G = 6.67\ 10^{-11}$ is the universal gravitational constant, ρ is the density and R is the radius of the planet. Substituting these last two for the values listed in Table 1, we can calculate the value of the surface gravity, g, for all planets.

Planet	R equatorial radius (km)	g surface gravity	ρ density
Mercury	2,439	0.378	5.4
Venus	6,052	0.894	5.3
Earth	6,378	1.000	5.5
Mars	3,397	0.379	3.9
Jupiter	71,492	2.540	1.3
Saturn	60,268	1.070	0.7
Uranus	25,559	0.800	1.2
Neptune	25,269	1.200	1.7

Table 6: Surface gravity and densities of the Solar System bodies.

Let's see a couple of examples:

$$g_{mercury} = \frac{4}{3}\pi\ G\ 5.4\ 2439 = 3.7$$

$$g_{venus} = \frac{4}{3}\pi\ G\ 5.3\ 6052 = 8.9$$

Similarly, we can calculate g for the rest of the planets. (Results are Mars: 3.7, Jupiter: 24.9, Saturn: 10.5, Uranus: 7.8 and Neptune: 11.8).

Model of bathroom scales

In this case, the goal of the model is to develop a set of 9 bathroom scales (8 planets and the Moon) so that students can simulate weighing themselves on each of the planets and the moon.

Since the process is the same for each planet, we will only describe one of them. The idea, essentially, is to open up a bathroom scale and replace the disk labeled with weights with another with weights calibrated for a particular planet.

1. First, we open the scale. In most scales, there are two springs that secure the base. Remember that we have to put it back together again (figures 13a and 13b).

2. Once open, the weight disk should be removed, either to be replaced, or drawn over with the appropriate planetary weights.
3. In the following table we have surface gravities of the moon and various planets of the Solar System. In

Planet	Gravity ($m \cdot s^{-2}$)	Gravity (T=1)
Moon	1.62	0.16
Mercury	3.70	0.37
Venus	8.87	0.86
Earth	9.80	1.00
Mars	3.71	0.38
Jupiter	23.12	2.36
Saturn	8.96	0.91
Uranus	8.69	0.88
Neptune	11.00	1,12

Table 7: Surface gravities for each Solar System body.

Fig. 13a: Surface gravities for each Solar System body.

Fig. 14: Solar System model with bathroom scales.

one column, they are listed in absolute values (m s^{-2}), and in the other in relative values with respect to terrestrial gravity. These values are the ones we will use to convert units of "terrestrial" weight to proportional units of weight on other planets.

4. Finally, we close the scale again, and can now see what we would weigh on one of the planets.

Models of craters

Most craters in the solar system are not volcanic but are the result falling meteoroids onto the surfaces of planets and satellites.

1. First, cover the floor with old newspapers, so that it doesn't get dirty.

2. Put a 2-3 cm layer of flour in a tray, distributing it with a strainer/sifter so that the surface is very smooth.

3. Put a layer of a few millimeters of cocoa powder above the flour with the help of a strainer/sifter (figure 15a).

4. From a height of about 2 meters, drop a projectile: a tablespoon of cocoa powder. The fall leaves marks similar to those of impact craters (figure 15b).

5. You may want to experiment with varying the height, type, shape, mass, etc. of the projectiles. In some cases, you can get even get a crater with a central peak.

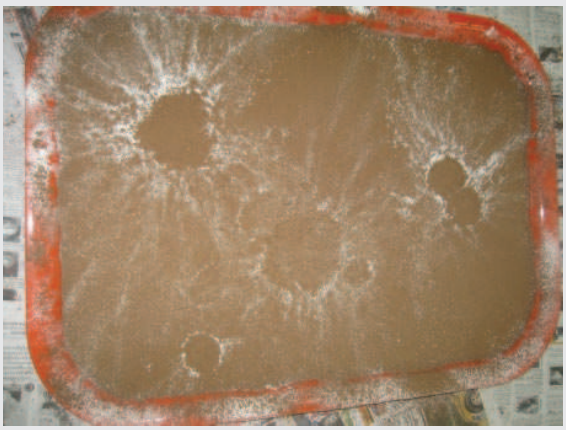


Fig. 15a: Simulating craters.
Fig. 15b: Resulting craters.

Model of escape velocities

If the launch speed of a rocket is not very large, the gravitational force of the planet itself makes the rocket fall back on its surface. If the launch speed is large enough, the rocket escapes from the planet's gravitational field. Let's calculate the speed above which a rocket can escape, ie the minimum launch speed or escape velocity.

Considering the formulas of uniformly accelerated motion,

$$e = ½\, at^2 + v_0\, t$$
$$v = at + v_0$$

if we replace the acceleration by g and we consider the initial velocity v_0 to be zero, we find that on the planet's surface, $R = ½\, gt^2$ and, moreover, $v = gt$. After removing the time variable, we find,

$$v = \sqrt{2gR}$$

where we can replace the values g and R by the values that are listed in the next table to calculate the escape velocity for each planet.

As an example, we calculate the escape velocities of some planets. For example:

For the Earth, $v_{earth} = \sqrt{2 \cdot g \cdot R} = (2\ 9.81\ 6378)^{1/2}$ km/s.

For the smallest planet, Mercury,
$v_{mercury} = (2\ 9.81\ 0.378\ 2439)^{1/2} = 4.2$ km/s

And for the greatest planet, Jupiter,
$v_{jupiter} = (2\ 9.81\ 0.378\ 2439)^{1/2} = 60.9$ km/s

It is clear that it is easier to launch a rocket from Mercury than from the Earth, but it is most difficult to launch a rocket on Jupiter, where the escape velocity is about 60 km/s.

Planet	R equatorial radius (km)	g reduced surface gravity
Mercury	2,439	0.378
Venus	6,052	0.894
Earth	6,378	1.000
Mars	3,397	0.379
Jupiter	71,492	2.540
Saturn	60,268	1.070
Uranus	25,559	0.800
Neptune	25,269	1.200

Table 8: Radius and surface gravities of Solar System bodies.

(To be able to compare the results, the accepted escape velocity for each body in the Solar System are the following: Mercury 4.3 km/s, Venus 10.3 km/s, Earth 11.2 km/s, Mars 5.0 km/s, Jupiter 59.5 km/s, Saturn 35.6 km/s, Uranus 21.2 km/s, Neptune 23.6 km/s. As we can see, our simple calculations give us acceptable results.)

Model of a rocket with an effervescent tablet

As an example of a rocket that can be launched safely in the classroom, we propose the following rocket, which uses an effervescent aspirin tablet as a propellant. We begin by cutting out the rocket model on the solid lines, and pasting on the dotted lines like in the photo.

We will use a plastic capsule, such as a film canister, making sure that the capsule can fit inside the cylinder of the rocket. Then, we put the three triangles as supports on the body of the rocket and finally, add the cone on the top of the cylinder (figures 16a, 16b, 16c, 16d, 17, 18, 19a, 19b, 19c).

Fig. 16a, 16b, 16c and 16d: The process in four pictures.

After constructing the rocket, we have to carry out the launch. For this, we will put water into the plastic capsule, up to about 1/3 of its height (about 1 cm). Add 1/4 of an effervescent tablet. Put the tape and the rocket above the capsule. After about 1 minute, the rocket takes off. Obviously we can repeat as many times as we would like (at least 3/4 of the aspirin tablet remains, so enjoy launching rockets!).

Fig. 17: Some rockets.

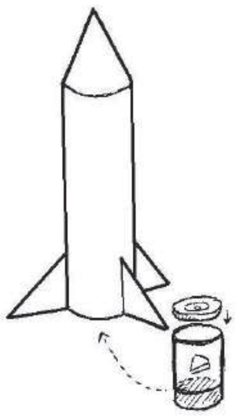


Fig. 18: Simplified scheme.

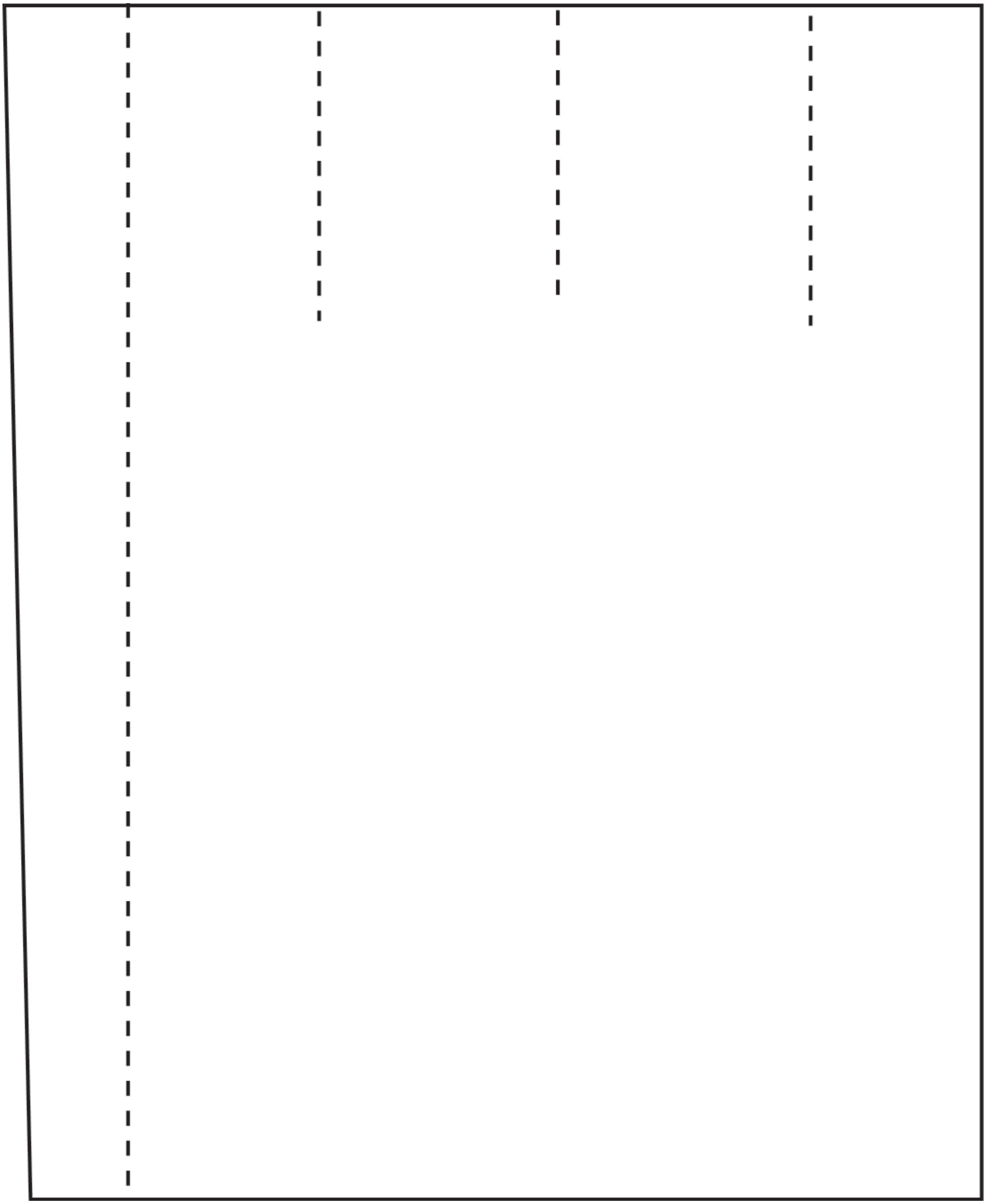
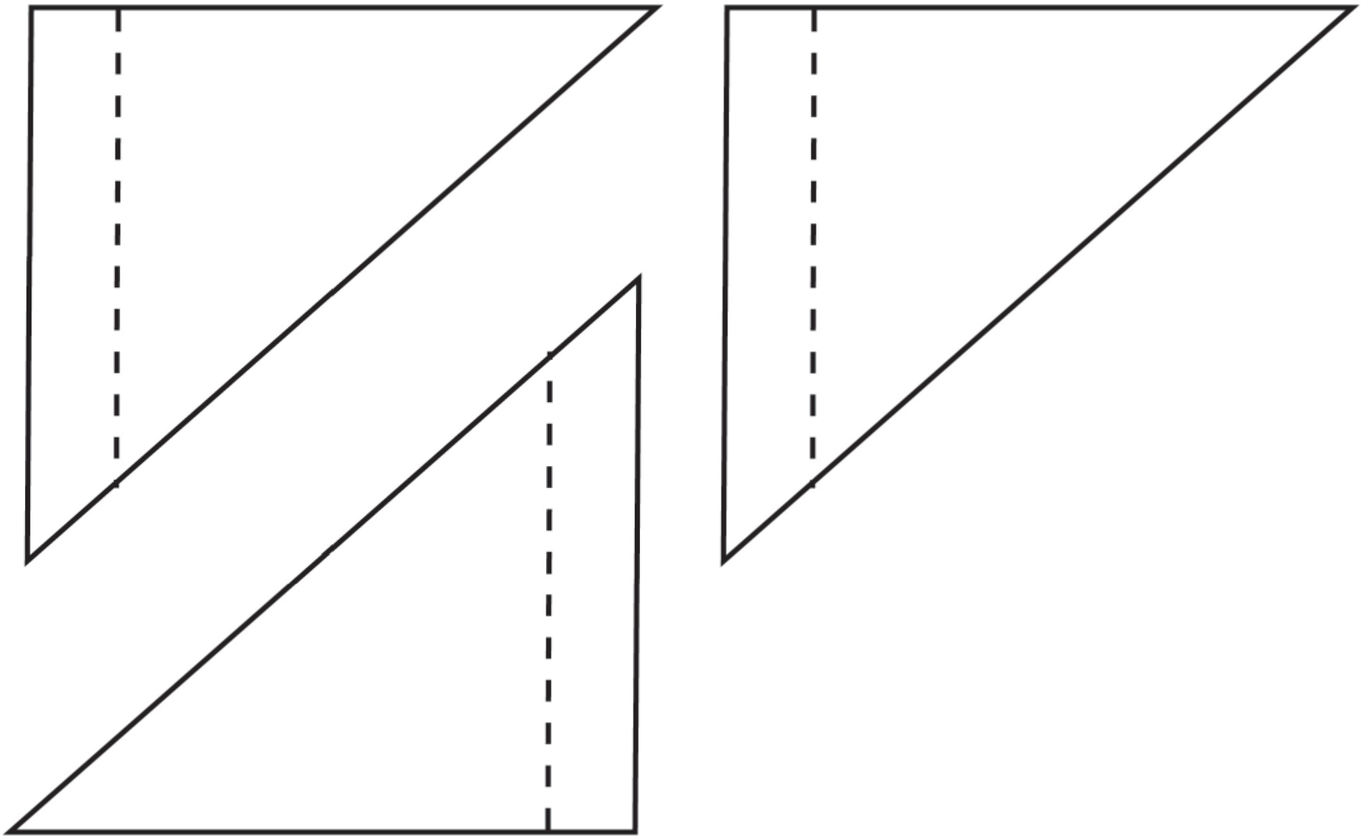


Fig. 19a: Body of the rocket. Paste the fins in the dotted zone.



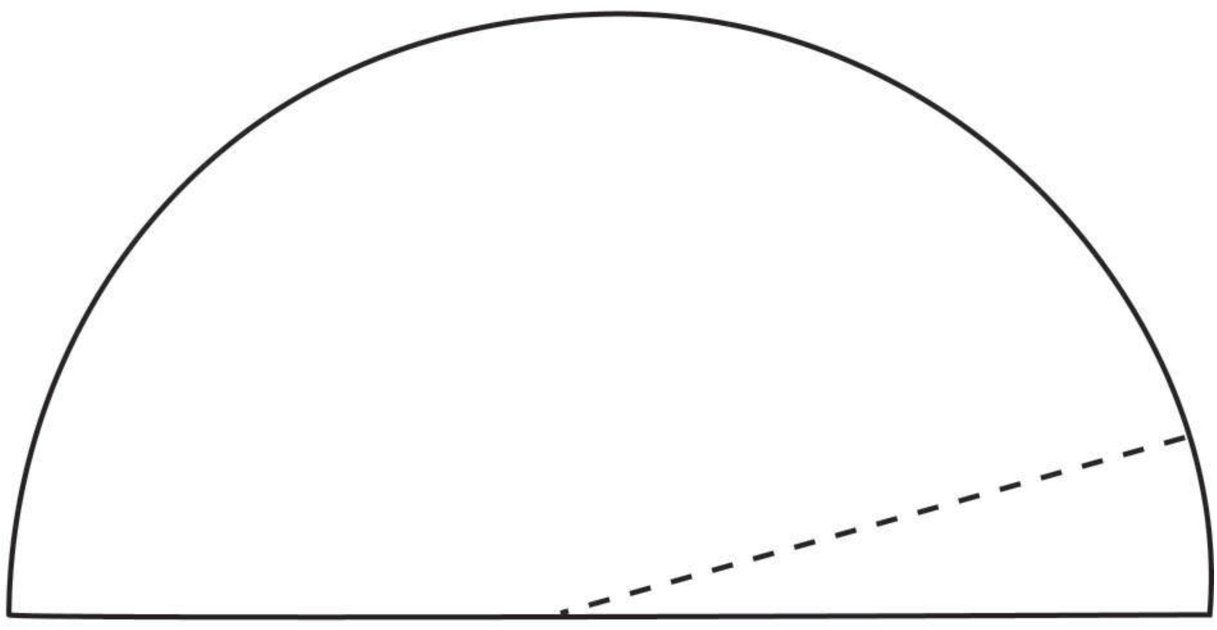


Fig. 19b: Model for the three fins. Fig.19c: Top cone of the rocket.

Models of exoplanetary systems

The Jet Propulsion Laboratory (NASA; http://planetquest.jpl.nasa.gov/) keeps a catalog of planetary objects discovered outside our own Solar System. There are more than 2000 planet candidates and more than 700 confirmed planets. They are called exoplanets (short for extrasolar planets; most are similar to or more massive than Jupiter, which is the largest planet in our Solar System. This is why we often compare the masses of extrasolar planets to the mass of Jupiter (1.9 10^{27} kg). Only a few of the exoplanets are similar in mass and size to the Earth, but this is likely due to an observational bias since the latest detection techniques are better at detecting massive objects.

In this section, we consider some examples of extrasolar planetary systems which have more than three known planets.

The nomenclature of exoplanets is simple. A letter is placed after the name of the star, beginning with the letter "b" for the first planet found in the system (e.g., 51 Pegasi b). The next planet detected in the system is labeled with the following letter of the alphabet such as, c, d, e, f, etc (e.g. 51 Pegasi c, 51 Pegasi d, 51 Pegasi e, 51 Pegasi f, etc).

Some exoplanets are very close to their central star, for example Gliese 876 with closer orbits that Mercury is from the sun. Others have more distant planets (HD 8799 has a planetary system with three planets about as far as Neptune is from the sun.) One possible way to display these data is to build scale models of the chosen planetary systems. This allows us to easily compare them with each other and with our Solar System.

Today we know that there are exoplanets around different types of stars. In 1992, radio astronomers announced the discovery of planets around pulsar PSR 1257+12. In 1995, the first detection of an exoplanet around a G-type star, 51 Pegasi, was announced, and since then exoplanets have been detected in orbit around: a red dwarf star (Gliese 876 in 1998), a giant star (Iota Draconis in 2001) , a brown dwarf star (2M1207 in 2004), a K-type star (HD40307 in 2008) and an A–type star (Fomalhaut in 2008), among others.

Planet name	Average distance, AU	Orbital period, days	Minimum mass*, Jupiter masses	Discovery date, year	Diameter **, km
Ups And b	0.059	4.617	0.69	1996	~Júpiter 124 000
Ups And c	0.83	241.52	1.98	1999	~Júpiter 176 000
Ups And d	2.51	1274.6	3.95	1999	~Júpiter 221 000
Gl 581 e	*0.03*	*3.149*	*0.006*	*2009*	*Terrestre 16000*
Gl 581 b	*0.04*	*5.368*	*0.049*	*2005*	*Terrestre 32 000*
Gl 581 c	*0.07*	*12.929*	*0.016*	*2007*	*Terrestre 22 000*
Gl 581 g (not confirmed)	*0.14*	*36.562*	*0.009*	*2005*	*Terrestre 18 000*
Gl 581 d	*0.22*	*68.8*	*0.024*	*2010*	*Terrestre 25000*
Gl 581 f (not confirmed)	*0.76*	*433*	*0.021*	*2010*	*Terrestre 24000*

Table 9: Extrasolar systems with multiple planets (three or more). Data from the *Extrasolar Planets Catalog*[2] (except the last column). * The radial velocity method only gives the minimum mass of the planet. ** The diameter shown in the last column has been calculated asuming that the planets density is equal to the density of Jupiter (1330 kg / m^3) for gaseous planets. For planets considered to be terrestrial, the diameter was calculated using the density of the Earth (5520 kg /m^3).

Planet name	Average distance, AU	Orbital period, years	Mass, Jupiter masses	Diameter, km
Mercury	0.3871	0.2409	0.0002	4,879
Venus	0.7233	0.6152	0.0026	12,104
Earth	1.0000	1.0000	0.0032	12,756
Mars	*1.5237*	*1.8809*	*0.0003*	*6,794*
Jupiter	*5.2026*	*11.8631*	*1*	*142,984*
Saturn	*9.5549*	*29.4714*	*0.2994*	*120,536*
Uranus	*19.2185*	*84.04*	*0.0456*	*51,118*
Neptune	*30.1104*	*164.80*	*0.0541*	*49,528*

Table 10: Solar System planets.

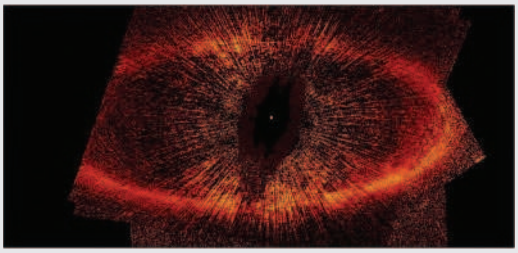


Fig. 20: Planet Fomalhaut b located in a debris disk, in an image of Fomalhaut taken by the Hubble Space Telescope (Photo:NASA).

Determination of the diameter of exoplanets

First, we will calculate the diameter of a couple of exoplanets included in Table 9.

We can achieve this goal by assuming that we know the density of the exoplanet. For our study, we consider that gaseous planets have the density of Jupiter and that terrestrial exoplanets have the same density as the planet Earth. By definition, the density of a body of mass m is given by : $\rho = m/V$

The mass m of the exoplanet appears in table 9, and the volume V can be obtained considering the planet to be a sphere:

$$V = \frac{4 \cdot \pi \cdot R^3}{3}$$

If we substitute this formula in the previous one, we can obtain the radius of the exoplanet:

$$R = \sqrt[3]{\frac{3m}{4\pi\rho}}$$

We suggest that the reader calculate the diameter of Gliese 581d (terrestrial exoplanet) assuming its density to be ρ = 5520 kg/m3 (the density of the Earth). Then repeat the calculation for a non-terrestrial exoplanet such as the first multiple planetary system that was discovered around a main sequence star, Upsilon Andromedae. This system consists of three planets, all of them similar to Jupiter: Ups b, c and d. Calculate their diameters assuming ρ = 1330 kg/m3 (the density of Jupiter) and compare the results with those in table 9.

Using these results and the average distance taken from table 9, we can produce a model in the next section.

Determination of the central star mass

Using the values of table 9 and Kepler's third law, we can determine the mass of the central star M. Kepler's third law states that for a planet with period P and an orbit of radius a, a^3/P^2 is a constant. We can show that this constant is the mass of the central star, expressed in solar masses. If we consider the motion of exoplanets around the star in a circular orbit of radius a, we can write, according to Newton's law of gravitation:

$$m \cdot \frac{v^2}{a} = \frac{G \cdot M \cdot m}{a^2}$$

For circular motion, the speed is

$$v^2 = \frac{G \cdot M}{a}$$

The period, for circular motion, is

$$P = \frac{2 \cdot \pi \cdot a}{v}$$

Then, when we introduce the value of v, we deduce:

$$P^2 = \frac{4 \cdot \pi^2 \cdot a^3}{G \cdot M}$$

And, for each exoplanet, using Kepler's third law,

$$\frac{a^3}{P^2} = \frac{G \cdot M}{4 \cdot \pi^2}$$

Writing the previous relation for the Earth's motion around the Sun, using P=1 year and a=1 AU, we deduce the following equation:

$$1 = \frac{G \cdot M_S}{4\,\pi^2}$$

Dividing the last two equalities, and taking the Sun's mass as unity, we obtain:

$$\frac{a^3}{P^2} = M$$

where a is the radius of the orbit (in AU), P is the period of revolution (in years). This relation allows us to determine the mass of the central star in units of solar masses.

Expressing the same relationship in diferent units, we can write:

$$M = 0{,}0395 \cdot 10^{-18} \frac{a^3}{P^2}\ M$$

where a is the radius of the orbit of the exoplanet (in km), P is the period of revolution of the exoplanet (in days) and M is the mass of the central star (in solar masses).

For example, calculate the mass of the stars Ups And and Gl 581 in solar masses (the result should be equal to 1.03 and 0.03 solar masses respectively).

Scale model of an exoplanetary system

First we choose the scale of the model. For distances, the appropriate scale is: 1 AU = 1 m. In this case all exoplanets can fit inside a typical classroom, as well as the first five planets in our Solar System. If the activity is carried out outside (e.g. in the school yard), we can build a complete model. A different scale needs to be used for the size of the planets, for example: 10,000 km = 0.5 cm. In this case, the largest planet (Jupiter) in our system will be 7 cm in diameter and the smallest (Mercury) will be 0.2 cm in size.

Now we can build the Solar System, the Upsilon Andromedae, and the Gliese 581 systems using the average distance values included in Tables 9 and 10, using the previously-calculated diameters.

In the past few years we have learned that the planetary systems configurations are diverse. Some of the exoplanets orbit around their stars much closer than any planet in our own Solar System orbits around the sun. Other exoplanets are closer to their parent star than Mercury is from the Sun. This means they are very hot. Another difference is that many large planets are close to their stars.

The inner part of the Solar System is populated by the small, rocky planets and the first of the gas giant planets, Jupiter, is at 5.2 AU from the Sun. These differences are believed to be mainly due to an observational bias. The radial velocity method for example is more sensitive when the planets are in smaller orbits and are more massive. But we may assume that most exoplanets have much larger orbits. It seems plausible that in most exoplanetary systems, there are one or two giant planets with orbits similar in size to those of Jupiter and Saturn.

We now consider the habitability of exoplanets. The habitable zone is the region around a star where a planet with sufficient atmospheric pressure can maintain liquid water on its surface. This is a conservative definition and it is restricted to life as we know it on Earth. Some planetary scientists have suggested to include equivalent zones around stars where other solvent compounds such as ammonia and methane could exist in stable liquid forms.

Rough calculations indicate that the solar system's habitable zone, where liquid water can exist (i.e. where the temperature ranges from 0 º to 100º C), ranges from 0.56 to 1.04 AU. The inner edge of this zone lies between the orbits of Mercury and Venus and the outer edge is just outside the orbit of Earth. Only two planets in the Solar System (Venus and Earth) are inside the habitable zone (the blue area in figure 21). As we know, only the Earth is inhabited, since Venus is too hot (but only because of a strong greenhouse effect on the planet).

It appears that Gliese 581d is an example of a terrestrial exoplanet within the habitable zone of its parent star, and it may be a potential candidate for extraterrestrial life.

Gliese 581 c, on the other hand, might be within the habitable zone of its host star. Its orbit lasts 13 days and it is situated 14 times closer to its star than the Earth lies from the Sun. Nevertheless, the smaller size of the star makes this distance favorable for the planet to harbor liquid water and to offer the possibility of life. Its radius is 1.5 times that of the Earth and this indicates that it is a rocky body. Its temperature ranges from 0º C to 40º C, which makes possible the existence of abundant liquid water. The problem is that it always presents the same face to the star. This evidence could suggest that the planet could be rocky like Earth or that it could be covered with oceans. Although, in contrast, some studies indicate that this planet is suffering from a significant greenhouse effect, like Venus.

Gliese 581 g is the first exoplanet, not yet confirmed, to be found within the habitable zone, with enough gravity to hold an atmosphere (3 to 4 times the mass of Earth) and the right temperature to shelter liquid water (-31º C to -12º C).

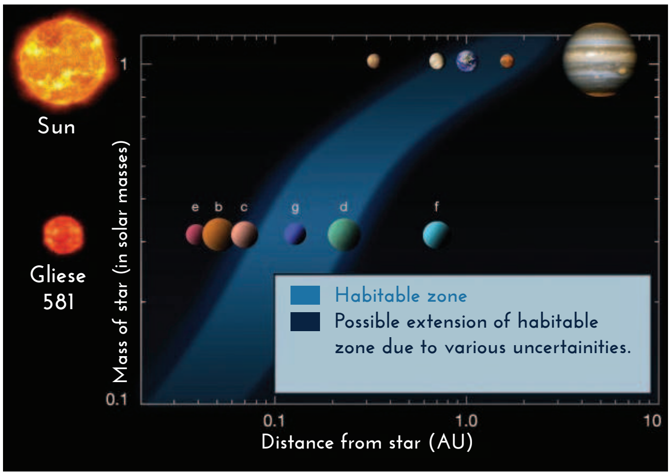


Fig. 21: The habitable zone. Comparison between the Solar System and the system of exoplanets in Gliese 581. The blue region indicates the zone where life as we konw it could exist.

Gliese 581 e is one of the smallest exoplanets ever discovered to date. Its mass is 1.7 the mass of the Earth, which makes it the smallest planet discovered and the closest in size to the planet Earth, although it has an orbit very close to its parent star at 0.03 AU. This fact makes it difficult to hold an atmosphere and puts it out of the habitable zone as the proximity of its parent star means that the temperatures are above 100 º C. At these temperatures, water is not in the liquid phase and life as we know it is not possible.

There are still many unanswered questions about the properties of exoplanets and there is much more to learn about their properties and characteristics.

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